CSE 250
Data Structures

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Day 14
Induction Review, Stacks and Queues
Textbook Ch. 15

Announcements

- PA2 is actually up now
    - Tests due 10/10
    - Full project due 10/17

Recap

QuickSort

- Divide and Conquer sorting algorithm like MergeSort
    - All of the work for Merge Sort happened during the combine step
    - QuickSort attempts to move the work to the divide step
- **Divide:** Move small elements to the left, and big elements to the right
- **Conquer:** Recursively call QuickSort on left and right halves
- **Combine:** …nothing

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QuickSort Review

Divide: Move *small* elements to the left and *big* elements to the right

How do we define what is *big* and what is *small*?

QuickSort Review

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How do we define what is *big* and what is *small*?

Pick a pivot value

QuickSort Review

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Pick a pivot value

[smaller than pivot], pivot, [larger than pivot]

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How do we define what is *big* and what is *small*?

Pick a pivot value

[smaller than pivot], pivot, [larger than pivot]

How do we pick a pivot?

QuickSort Review

[4, 1, 8, 13, 12, 6, 2, 14, 7, 9, 3, 5, 11, 10, 15]

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QuickSort Review

If our pivot was the median value, then our list would be split in half by the divide step, resulting in the same runtime as MergeSort $O(n\log(n))$.

But finding the median value is expensive…(it also costs $n\log(n)$).

So what if we pick one randomly instead?

Expected Value

If I roll a 6-sided die, the probability of a particular side being rolled is ⅙

If *X* is a random variable representing this die roll, then the expected value of *X* is:

$$E[X] = \frac{1}{6} \cdot 1 + \frac{1}{6} \cdot 2 + \frac{1}{6} \cdot 3 + \frac{1}{6} \cdot 4 + \frac{1}{6} \cdot 5 + \frac{1}{6} \cdot 6$$

$$E[X] = \sum_{i=1}^{6} \frac{1}{6} i = 3.5$$

Expected Value

If I roll a 20-sided die, the probability of a particular side being rolled is 1/20

If *X* is a random variable representing this die roll, then the expected value of *X* is:

$$E[X] = \frac{1}{20} \cdot 1 + \frac{1}{20} \cdot 2 + \ldots + \frac{1}{20} \cdot 20 = \sum_{i=1}^{20} \frac{1}{20} i$$

Expected Value

If I roll an *n*-sided die, the probability of a particular side being rolled is 1/*n*

If *X* is a random variable representing this die roll, then the expected value of *X* is:

$$E[X] = \frac{1}{n} \cdot 1 + \frac{1}{n} \cdot 2 + ... + \frac{1}{n} \cdot n = \sum_{i=1}^{n} \frac{1}{n} i$$

$$E[X] = \sum_{i} P_i \cdot X_i$$

QuickSort Review

Picking a pivot value randomly from the n elements of our sequence is the same as rolling an n-sided die.

There is a $1/n$ probability in any particular value being selected.

$X = k$ means that X is the kth largest value, and the expected value of X corresponds to the median value.

QuickSort Review

$$T(n) = \begin{cases} \Theta(1) & \textbf{if } n \leq 1 \\ T(0) + T(n-1) + \Theta(n) & \textbf{if } n > 1 \wedge X = 1 \\ T(1) + T(n-2) + \Theta(n) & \textbf{if } n > 1 \wedge X = 2 \\ T(2) + T(n-3) + \Theta(n) & \textbf{if } n > 1 \wedge X = 3 \\ .. & \\ T(n-2) + T(1) + \Theta(n) & \textbf{if } n > 1 \wedge X = n-1 \\ T(n-1) + T(0) + \Theta(n) & \textbf{if } n > 1 \wedge X = n \end{cases}$$

QuickSort Review

$$E[T(n)] = \begin{cases} \Theta(1) & \textbf{if } n \leq 1 \\ E[T(X-1) + T(n-X)] + \Theta(n) & \textbf{otherwise} \end{cases}$$

QuickSort Review

$$E[T(n)] = \begin{cases} \Theta(1) & \textbf{if } n \leq 1 \\ \boxed{E[T(X-1) + T(n-X)]} + \Theta(n) & \textbf{otherwise} \end{cases}$$

Expected value of two independent events can be split up

QuickSort Review

$$E[T(n)] = \begin{cases} \Theta(1) & \textbf{if } n \leq 1 \\ E[T(X-1)] + E[T(n-X)] + \Theta(n) & \textbf{otherwise} \end{cases}$$

QuickSort Review

$$E[T(n)] = \begin{cases} \Theta(1) & \textbf{if } n \leq 1 \\ \boxed{E[T(X-1)] + E[T(n-X)]} + \Theta(n) & \textbf{otherwise} \end{cases}$$

How are these two terms related?

QuickSort Review

$$E[T(X-1)]$$

QuickSort Review

$$E[T(X-1)]$$

$$= \sum_{i=1}^{n} P_i \cdot T(X_i - 1)$$

QuickSort Review

$$E[T(X-1)]$$

$$= \sum_{i=1}^{n} P_i \cdot T(X_i - 1)$$

$$= \sum_{i=1}^{n} \frac{1}{n} \cdot T(i-1)$$

QuickSort Review

$$E[T(X-1)]$$

$$= \sum_{i=1}^{n} P_i \cdot T(X_i - 1)$$

$$= \sum_{i=1}^{n} \frac{1}{n} \cdot T(i-1)$$

$$= \sum_{i=1}^{n} \frac{1}{n} \cdot T(n-i)$$

QuickSort Review

$$E[T(X-1)]$$

$$= \sum_{i=1}^{n} P_i \cdot T(X_i - 1)$$

$$= \sum_{i=1}^{n} \frac{1}{n} \cdot T(i-1)$$

$$= \sum_{i=1}^{n} \frac{1}{n} \cdot T(n-i) = E[T(n-X)]$$

QuickSort Review

$$E[T(X-1)]$$

$$= \sum_{i=1}^{n} P_i \cdot T(X_i - 1)$$

$$= \sum_{i=1}^{n} \frac{1}{n} \cdot T(i-1)$$

$$= \sum_{i=1}^{n} \frac{1}{n} \cdot T(n-i) \quad = E[T(n-X)]$$

They are equivalent!!

QuickSort Review

$$E[T(n)] = \begin{cases} \Theta(1) & \textbf{if } n \leq 1 \\ 2E[T(X-1)] + \Theta(n) & \textbf{otherwise} \end{cases}$$

QuickSort Review

$$E[T(n)] = \begin{cases} \Theta(1) & \textbf{if } n \leq 1 \\ \boxed{2E[T(X-1)]} + \Theta(n) & \textbf{otherwise} \end{cases}$$

Each T(X-1) is independent, so the expected values can be split out

QuickSort Review

$$E[T(n)] = \begin{cases} \Theta(1) & \textbf{if } n \leq 1 \\ \frac{2}{n}\left(\sum_{i=0}^{n-1} E[T(i)]\right) + \Theta(n) & \textbf{otherwise} \end{cases}$$

Inductive Proof

Back to Induction

Hypothesis: $E[T(n)] \in O(n \log(n))$

Base Case

Base Case: $E[T(2)] \leq c\ (2 \log(2))$

Base Case

Base Case: $E[T(2)] \le c\,(2\log(2))$

$$2 \cdot E_i[T(i - 1)] + 2c_1 \le 2c$$

Base Case

Base Case: $E[T(2)] \leq c\,(2\log(2))$

$$2 \cdot E_i[T(i - 1)] + 2c_1 \leq 2c$$

$$2 \cdot (T(0)/2 + T(1)/2) + 2c_1 \leq 2c$$

Base Case

Base Case: $E[T(2)] \leq c\,(2 \log(2))$

$$2 \cdot E_i[T(i - 1)] + 2c_1 \leq 2c$$

$$\cancel{2} \cdot (T(0)/\cancel{2} + T(1)/\cancel{2}) + 2c_1 \leq 2c$$

$$T(0) + T(1) + 2c_1 \leq 2c$$

Base Case

Base Case: $E[T(2)] \leq c\ (2 \log(2))$

$$2 \cdot E_i[T(i - 1)] + 2c_1 \leq 2c$$

$$2 \cdot (T(0)/2 + T(1)/2) + 2c_1 \leq 2c$$

$$T(0) + T(1) + 2c_1 \leq 2c$$

$$2c_0 + 2c_1 \leq 2c$$

Base Case

Base Case: $E[T(2)] \leq c\ (2 \log(2))$

$2 \cdot E_i[T(i - 1)] + 2c_1 \leq 2c$

$2 \cdot (T(0)/2 + T(1)/2) + 2c_1 \leq 2c$

$T(0) + T(1) + 2c_1 \leq 2c$

$2c_0 + 2c_1 \leq 2c$

True for any $c \geq c_0 + c_1$

Inductive Case

Assume: $E[T(n')] \leq c\,(n' \log(n'))$ for **all** $n' < n$

Show: $E[T(n)] \leq c\,(n \log(n))$

Inductive Case

Assume: $E[T(n')] \leq c\,(n' \log(n'))$ for **all** $n' < n$

Show: $E[T(n)] \leq c\,(n \log(n))$

$$\frac{2}{n}\left(\sum_{i=0}^{n-1} E[T(i)]\right) + c_1 \leq cn\log(n)$$

Inductive Case

Assume: $E[T(n')] \leq c\,(n' \log(n'))$ for **all** $n' < n$

Show: $E[T(n)] \leq c\,(n \log(n))$

Our i here is always less than n, so we can use our assumption to substitute

$$\frac{2}{n}\left(\sum_{i=0}^{n-1} E[T(i)]\right) + c_1 \leq cn\log(n)$$

$$\frac{2}{n}\left(\sum_{i=0}^{n-1} ci\log(i)\right) + c_1 \leq cn\log(n)$$

Inductive Case

Assume: $E[T(n')] \leq c\,(n' \log(n'))$ for **all** $n' < n$

Show: $E[T(n)] \leq c\,(n \log(n))$

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$$\frac{2}{n}\left(\sum_{i=0}^{n-1} ci\log(i)\right) + c_1 \leq cn\log(n)$$

$$c\frac{2}{n}\left(\sum_{i=0}^{n-1} i\log(n)\right) + c_1 \leq cn\log(n)$$

Inductive Case

$$c\frac{2}{n}\left(\sum_{i=0}^{n-1} i\log(n)\right) + c_1 \leq cn\log(n)$$

Inductive Case

$$c\frac{2}{n}\left(\sum_{i=0}^{n-1} i\log(n)\right) + c_1 \leq cn\log(n)$$

$$c\frac{2\log(n)}{n}\left(\sum_{i=0}^{n-1} i\right) + c_1 \leq cn\log(n)$$

Inductive Case

$$c\frac{2}{n}\left(\sum_{i=0}^{n-1} i\log(n)\right) + c_1 \leq cn\log(n)$$

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Inductive Case

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$$cn\log(n) - c\log(n) + c_1 \leq cn\log(n)$$

Inductive Case

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$$cn\log(n) - c\log(n) + c_1 \leq cn\log(n)$$

$$c_1 \leq c\log(n)$$

QuickSort

So…is QuickSort $O(n \log(n))$...?

No!

What guarantees do you get?

If $f(n)$ is a Tight Bound

The algorithm always runs in $cf(n)$ steps

If $f(n)$ is a Worst-Case Bound

The algorithm always runs in at most $cf(n)$

If $f(n)$ is an Amortized Worst-Case Bound

n invocations of the algorithm **always** run in $cnf(n)$ steps

If $f(n)$ is an Average Bound

…we don't have any guarantees

`mutable.Seq` ADT

`mutable.IndexedSeq` (ie Array)
Efficiency apply(), update()

`mutable.Buffer` (ie ArrayBuffer, ListBuffer)
Efficiency apply(), update(), append()

Stacks

A stack of objects on top of one another

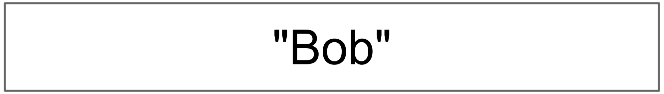
Push Put a new object on top of the stack

Pop Remove the object on top of the stack

Top Peek at what's on top of the stack

Stacks

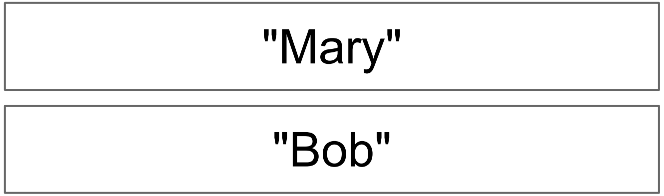
```
s.push("Bob")
```



Stacks

```
s.push("Bob")

s.push("Mary")
```

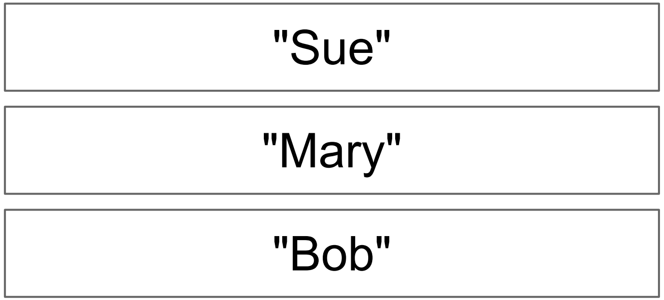


Stacks

```
s.push("Bob")

s.push("Mary")

s.push("Sue")
```



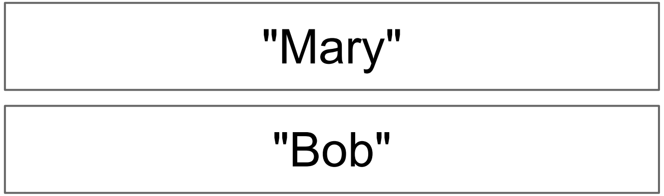
Stacks

```
s.push("Bob")

s.push("Mary")

s.push("Sue")

s.pop()
```



Stacks

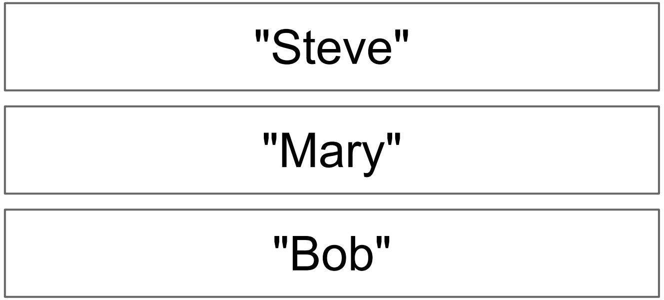
```
s.push("Bob")

s.push("Mary")

s.push("Sue")

s.pop()

s.push("Steve")
```



Stacks

```
s.push("Bob")

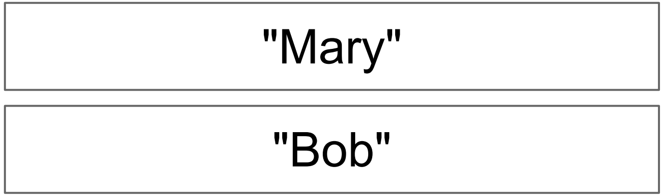
s.push("Mary")

s.push("Sue")

s.pop()

s.push("Steve")

s.pop()
```



Stacks in Practice

- Storing function variables in a "call stack"
- Certain types of parsers ("context free")
- Backtracking search
- Reversing Sequences

Stacks

```
trait Stack[A] {
  def push(element: A): Unit
  def top: A
  def pop: A
}
```

Stacks

```
class ListStack[A] extends Stack[A] {
      val _store = new SinglyLinkedList()

      def push(element: A): Unit =
        _store.prepend(element)

      def top: A =
        _store.head

      def pop: A =
        _store.remove(0)
    }
```

Stacks

```
class ListStack[A] extends Stack[A] {
      val _store = new SinglyLinkedList()

      def push(element: A): Unit =
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What is the runtime?

Stacks

```
class ListStack[A] extends Stack[A] {
      val _store = new SinglyLinkedList()

      def push(element: A): Unit =
        _store.prepend(element)

      def top: A =
        _store.head

      def pop: A =
        _store.remove(0)
    }
```

push: $\Theta(1)$

top: $\Theta(1)$

pop: $\Theta(1)$

What is the runtime?

Stacks

```
class ArrayBufferStack[A] extends Stack[A] {
      val _store = new ArrayBuffer()

      def push(element: A): Unit =
        _store.append(element)

      def top: A =
        _store.last

      def pop: A =
        _store.remove(store.length-1)
    }
```

Stacks

```
class ArrayBufferStack[A] extends Stack[A] {
      val _store = new ArrayBuffer()

      def push(element: A): Unit =
        _store.append(element)

      def top: A =
        _store.last

      def pop: A =
        _store.remove(store.length-1)
    }
```

What is the runtime?

Stacks

```
class ArrayBufferStack[A] extends Stack[A] {
      val _store = new ArrayBuffer()

      def push(element: A): Unit =
        _store.append(element)

      def top: A =
        _store.last

      def pop: A =
        _store.remove(store.length-1)
    }
```

Amortized $O(1)$

$\Theta(1)$

$\Theta(1)$

What is the runtime?

Stacks in Scala

Scala's **`Stack`** implementation is based on **`ArrayBuffer`**; Keeping memory together is worth the overhead of amortized $O(1)$.

Queue

Outside of the US, "queueing" is lining up, ie at Starbucks

Enqueue Put a new object at the end of the queue

Dequeue Remove the next object in the queue

Head Peek at the next object in the queue

Queue

Front

Back

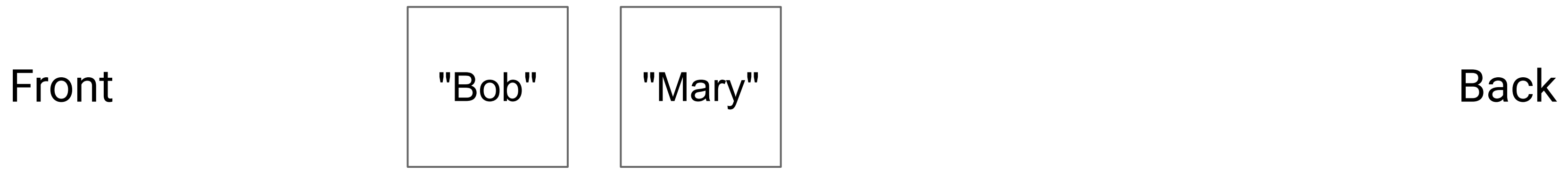
Queue

`enqueue("Bob")`

Front "Bob" Back

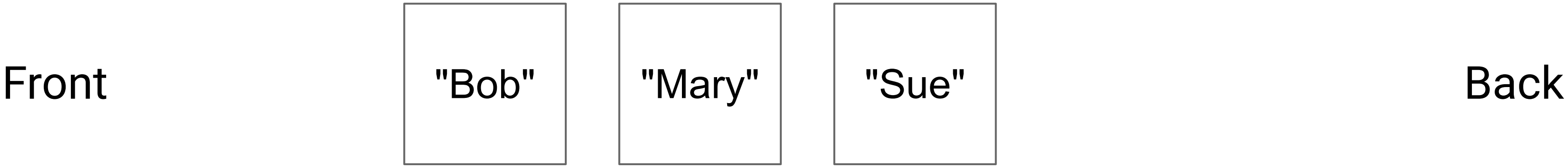
Queue

```
enqueue("Bob")
enqueue("Mary")
```



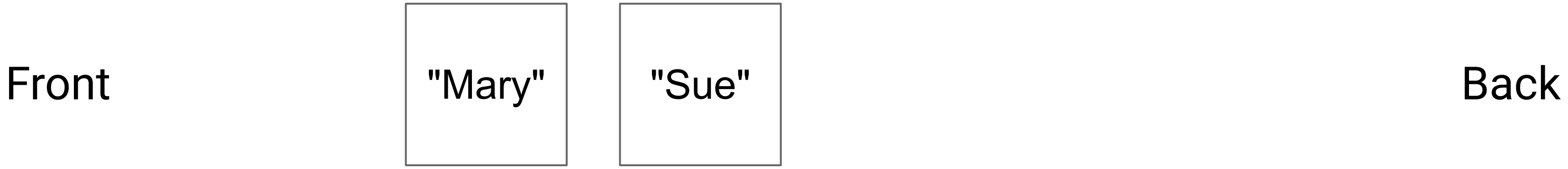
Queue

```
enqueue("Bob")
enqueue("Mary")
enqueue("Sue")
```



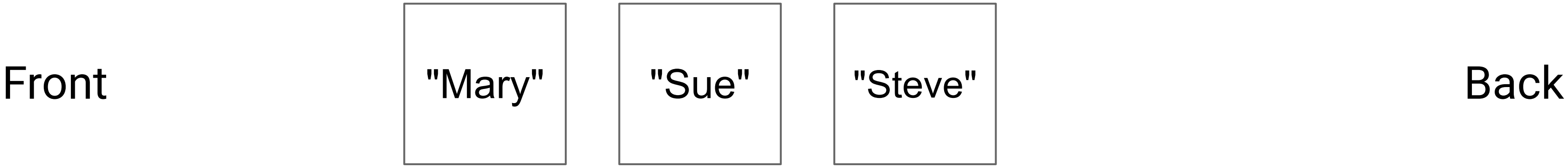
Queue

```
enqueue("Bob")
enqueue("Mary")
enqueue("Sue")
dequeue()
```



Queue

```
enqueue("Bob")
enqueue("Mary")
enqueue("Sue")
dequeue()
enqueue("Steve")
```



Queue

```
enqueue("Bob")
enqueue("Mary"
enqueue("Sue")
dequeue()
enqueue("Steve")
dequeue()
```

Queues vs Stacks

Queue First in, First Out (FIFO)

Statcks Last in, First Out (LIFO / FILO)

Queues in Practice

- Delivering network packets, emails, twitter/tiktok/instagram
- Scheduling CPU cycles
- Deferring long-running tasks

Queues

```
trait Queue[A] {
  def enqueue(element: A): Unit
  def dequeue: A
  def head: A
}
```

Queues

```
class ListQueue[A] extends Queue[A] {
      val _store = new DoublyLinkedList()

      def enqueue(element: A): Unit =
        _store.append(element)

      def head: A =
        _store.head

      def dequeue: A =
        _store.remove(0)
    }
```

Queues

```
class ListQueue[A] extends Queue[A] {
      val _store = new DoublyLinkedList()

      def enqueue(element: A): Unit =
        _store.append(element)

      def head: A =
        _store.head

      def dequeue: A =
        _store.remove(0)
    }
```

What is the runtime?

Queues

```
class ListQueue[A] extends Queue[A] {
      val _store = new DoublyLinkedList()

      def enqueue(element: A): Unit =
        _store.append(element)

      def head: A =
        _store.head

      def dequeue: A =
        _store.remove(0)
    }
```

enqueue: $\Theta(1)$

head: $\Theta(1)$

dequeue: $\Theta(1)$

What is the runtime?

Queues

Thought Experiment: How can we use an array to build a queue?