CSE 250
Data Structures

Dr. Eric Mikida
epmikida@buffalo.edu

Dr. Oliver Kennedy
okennedy@buffalo.edu

212 Capen Hall

Day 17
Graph Exploration
Textbook Ch. 15.3

Edge List Summary

- `addEdge, addVertex`: $O(1)$
- `removeEdge`: $O(1)$
- `removeVertex`: $O(m)$
- `vertex.incidentEdges`: $O(m)$
- `vertex.edgeTo`: $O(m)$
- **Space Used:** $O(n) + O(m)$

Adjacency List Summary

- `addEdge, addVertex`: $O(1)$
- `removeEdge`: $O(1)$
- `removeVertex`: $O(\text{deg}(\text{vertex}))$
- `vertex.incidentEdges`: $O(\text{deg}(\text{vertex}))$
- `vertex.edgeTo`: $O(\text{deg}(\text{vertex}))$
- **Space Used:** $O(n) + O(m)$

Adjacency Matrix Summary

- `addEdge, removeEdge`: $O(1)$
- `addVertex, removeVertex`: $O(n^2)$
- `vertex.incidentEdges`: $O(n)$
- `vertex.edgeTo`: $O(1)$
- **Space Used:** $O(n^2)$

So…what do we do with our graphs?

Connectivity Problems

Given graph ***G***:

Connectivity Problems

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- Is vertex ***u*** **adjacent** to vertex ***v***?

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- Is vertex ***u*** **adjacent** to vertex ***v***?
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- Is vertex ***u*** **connected** to vertex ***v*** via some path?
- Which vertices are **connected** to vertex ***v***?

Connectivity Problems

Given graph ***G***:

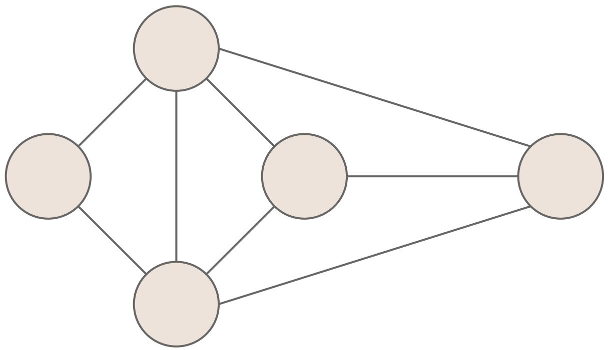
- Is vertex ***u*** **adjacent** to vertex ***v***?
- Is vertex ***u*** **connected** to vertex ***v*** via some path?
- Which vertices are **connected** to vertex ***v***?
- What is the **shortest path** from vertex ***u*** to vertex ***v***?

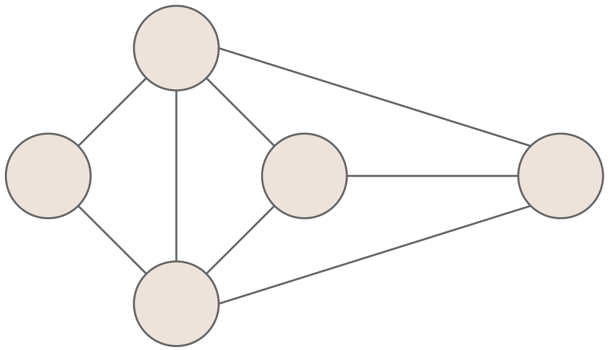
A few more definitions

A **subgraph, *S*,** of a graph ***G*** is a graph where:

S's vertices are a subset of ***G***'s vertices

S's edges are a subset of ***G***'s edges





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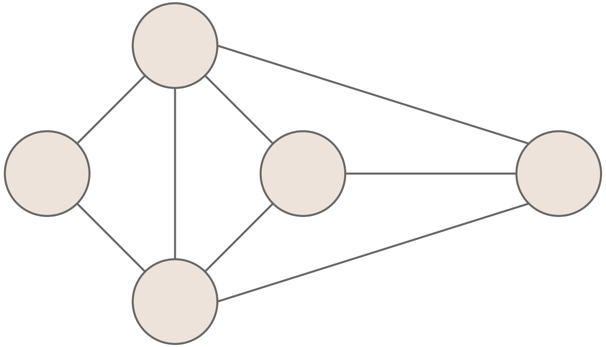
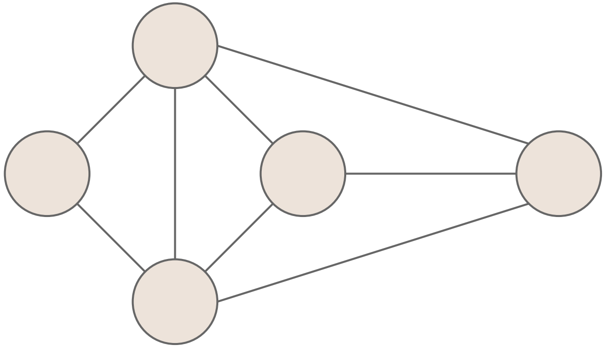


A **spanning subgraph** of ***G*...**

Is a subgraph of ***G***

Contains all of ***G***'s vertices



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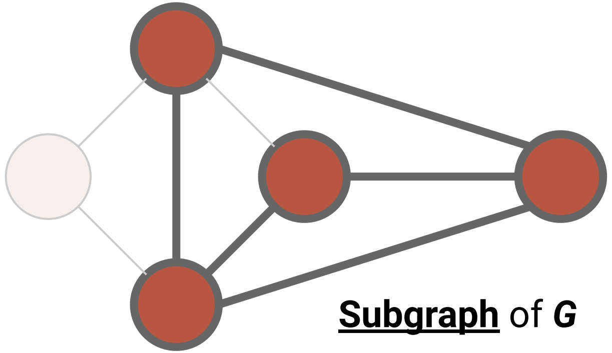

Subgraph of ***G***

A **<u>spanning subgraph</u>** of ***G*...**

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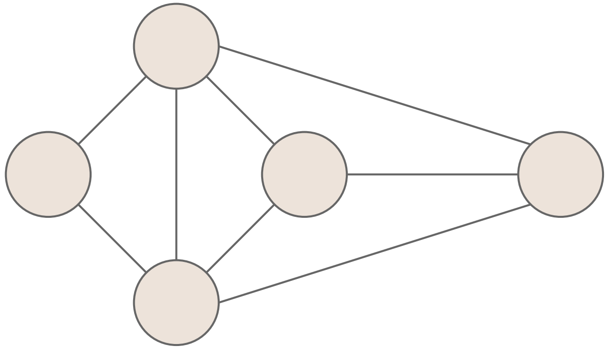

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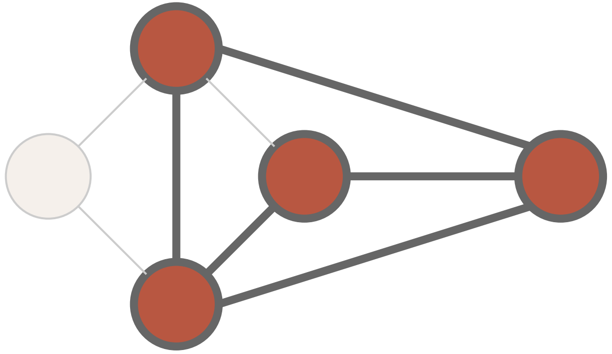

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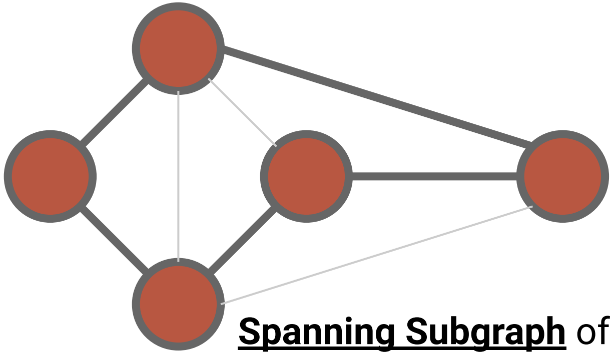
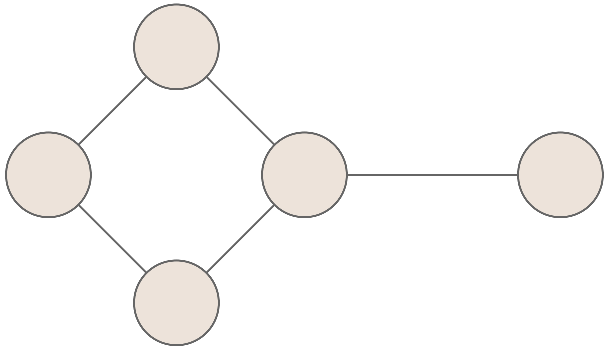


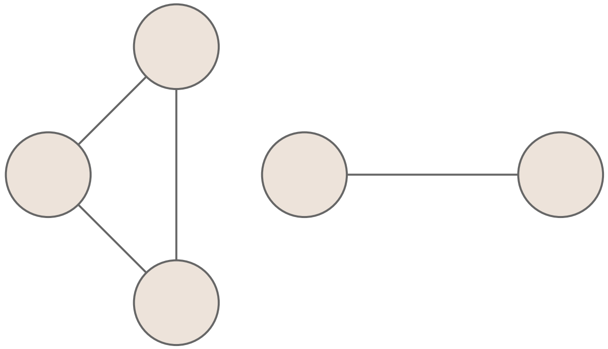
<u>Spanning Subgraph</u> of ***G***

A few more definitions

A graph is **connected**…

If there is a path between every pair of vertices





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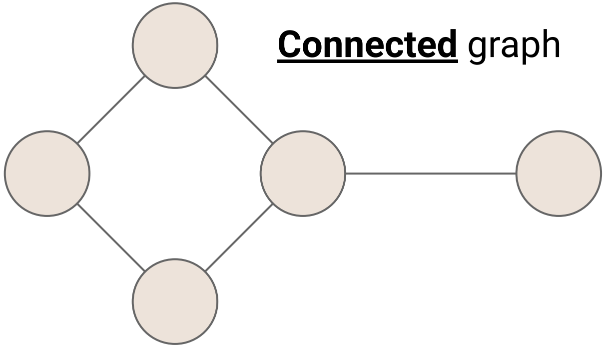
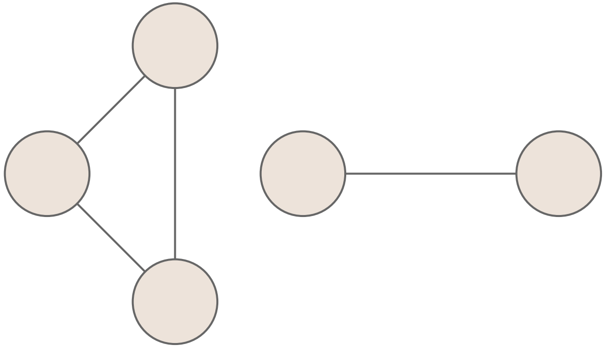


Connected graph



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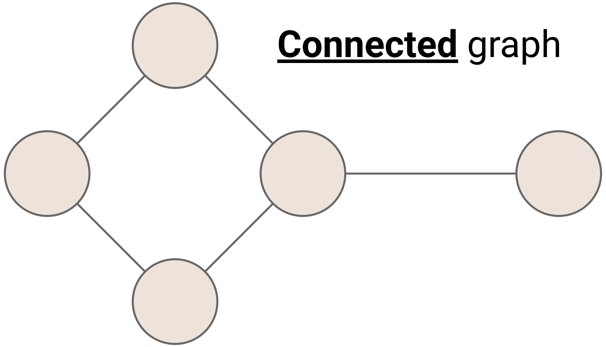

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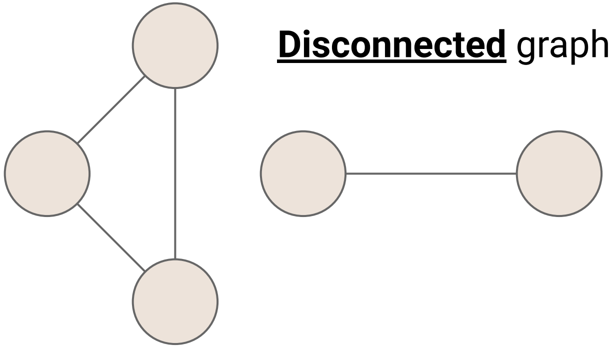


Disconnected graph

A few more definitions

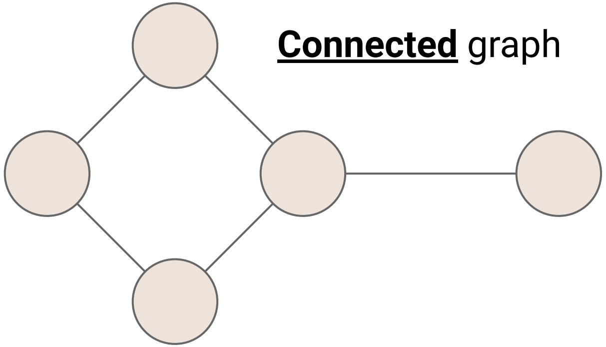
A graph is **connected**…

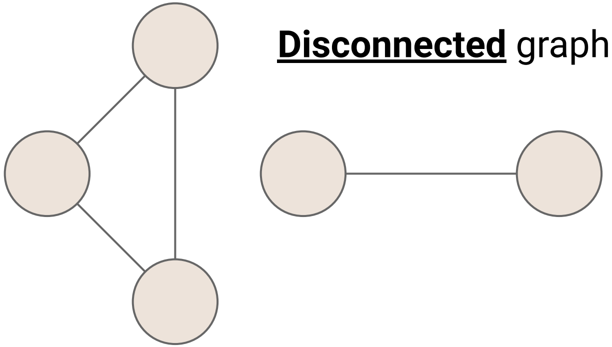
If there is a path between every pair of vertices

A **connected component** of ***G*…**

Is a maximal connected subgraph of ***G***

- "maximal" means you can't add a new vertex without breaking the property
- Any subset of ***G***'s edges that connect the subgraph are fine





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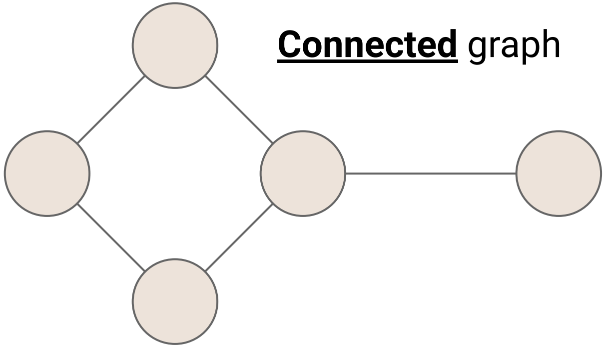

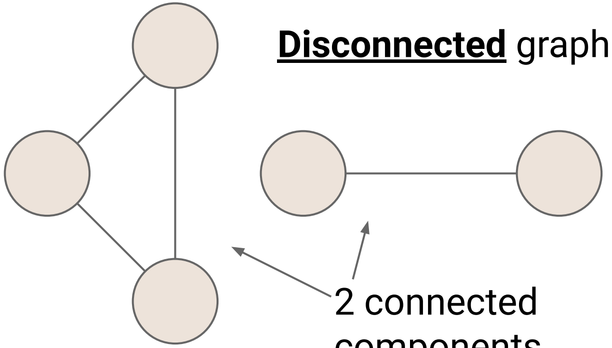

A few more definitions

A **free tree** is an undirected graph ***T*** such that…

There is exactly one simple path between any two nodes

- ***T*** is connected
- ***T*** has no cycles

A few more definitions

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A **rooted tree** is a directed graph ***T*** such that…

One vertex of ***T*** is the **root**

There is exactly one simple path from the root to every other vertex in the graph

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A **rooted tree** is a directed graph ***T*** such that…

One vertex of ***T*** is the **root**

There is exactly one simple path from the root to every other vertex in the graph

A (free/rooted) **forest** is a graph ***F*** such that…

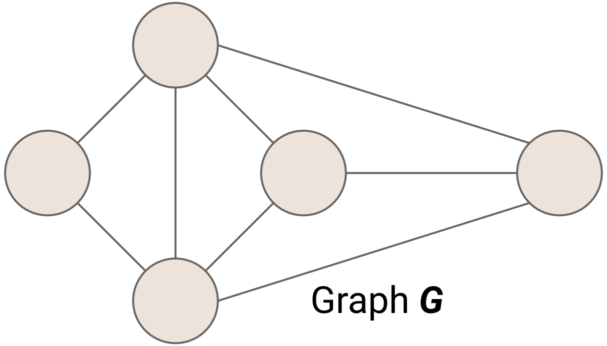
Every connected component is a tree

A few more definitions

A **spanning tree** of a connected graph…

…Is a spanning subgraph that is a tree

…It is not unique unless the graph is a tree



Graph ***G***

A few more definitions

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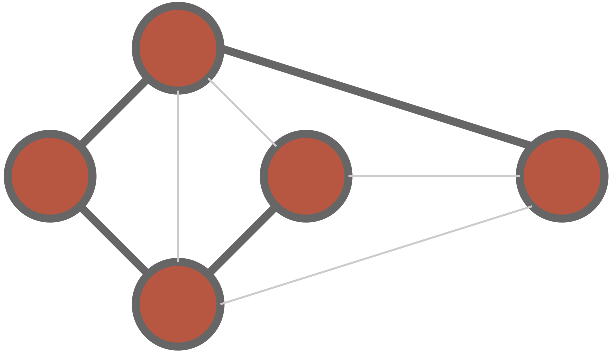
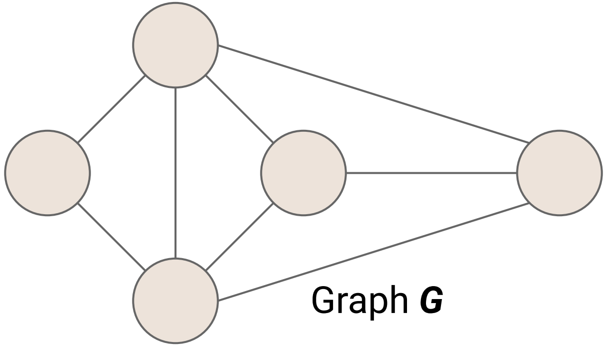
A **Spanning Tree** of ***G***





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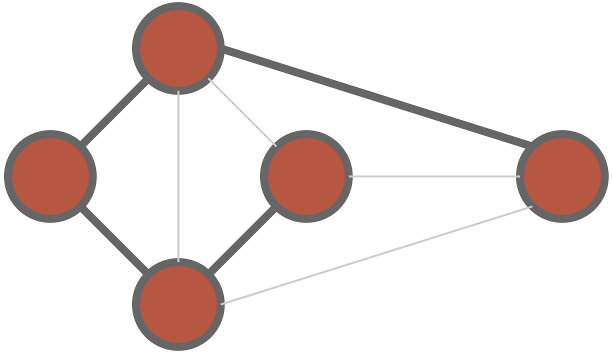
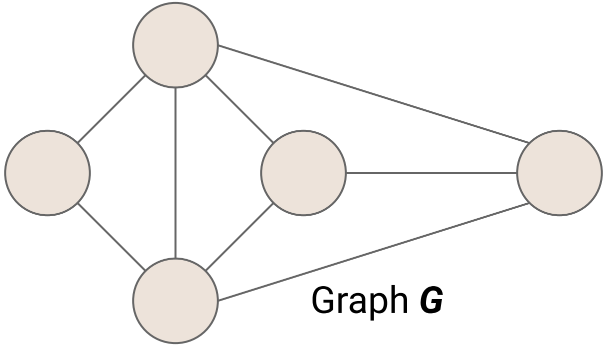
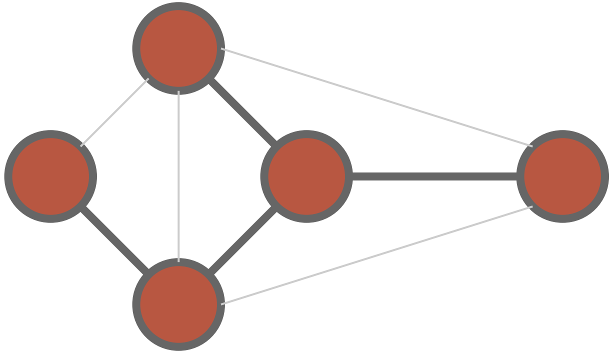




Graph ***G***

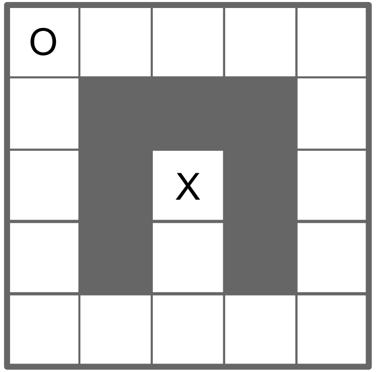
Another **Spanning Tree** of ***G***



Now back to the question…Connectivity

Back to Mazes

How could we represent our maze as a graph?



Back to Mazes

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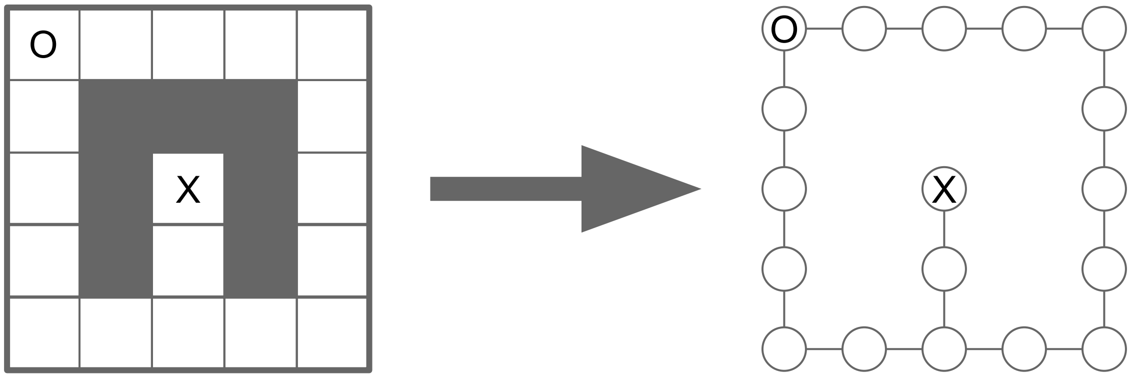

Recall

Searching the maze with a stack

We try every path, one at a time, following it as far as we can

…then backtrack and try another

Recall

Searching the maze with a stack (Depth-First Search)

We try every path, one at a time, following it as far as we can

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Recall

Searching the maze with a stack (Depth-First Search)

We try every path, one at a time, following it as far as we can

…then backtrack and try another

Searching with a queue?

TBD…

Depth-First Search

Primary Goals

- Visit every vertex in graph ***G* = (*V*,*E*)**
- Construct a spanning tree for every connected component

Depth-First Search

Primary Goals

- Visit every vertex in graph $G = (V,E)$
- Construct a spanning tree for every connected component
  - **Side Effect:** Compute connected components

Depth-First Search

Primary Goals

- Visit every vertex in graph ***G* = (*V*,*E*)**
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  - **Side Effect:** Compute a path between all connected vertices

Depth-First Search

<u>Primary Goals</u>

- Visit every vertex in graph ***G* = (*V*,*E*)**
- Construct a spanning tree for every connected component
  - **Side Effect:** Compute connected components
  - **Side Effect:** Compute a path between all connected vertices
  - **Side Effect:** Determine if the graph is connected

Depth-First Search

Primary Goals

- Visit every vertex in graph ***G* = (*V*,*E*)**
- Construct a spanning tree for every connected component
  - **Side Effect:** Compute connected components
  - **Side Effect:** Compute a path between all connected vertices
  - **Side Effect:** Determine if the graph is connected
  - **Side Effect:** Identify cycles

Depth-First Search

Primary Goals

- Visit every vertex in graph ***G* = (*V*,*E*)**
- Construct a spanning tree for every connected component
  - **Side Effect:** Compute connected components
  - **Side Effect:** Compute a path between all connected vertices
  - **Side Effect:** Determine if the graph is connected
  - **Side Effect:** Identify cycles
- Complete in time $O(|V| + |E|)$

Depth-First Search

DFS

Input: Graph ***G* = (*V*,*E*)**

Output: Label every edge as:

- Spanning Edge: Part of the spanning tree
- Back Edge: Part of a cycle

Depth-First Search

DFS

Input: Graph $G = (V,E)$

Output: Label every edge as:

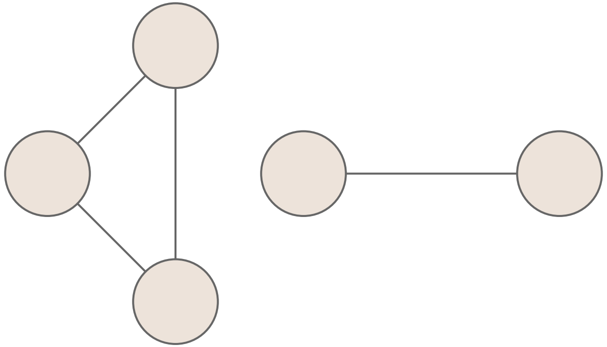
- Spanning Edge: Part of the spanning tree
- Back Edge: Part of a cycle

DFSOne

Input: Graph $G = (V,E)$, start vertex $v \in V$

Output: Label every edge in v's connected component

Depth-First Search



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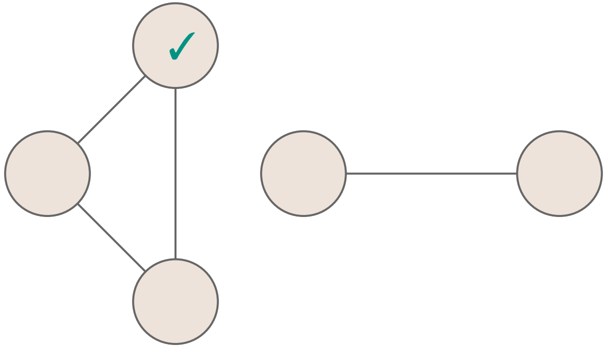

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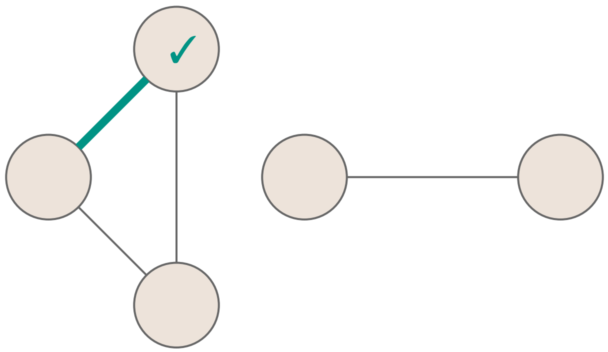

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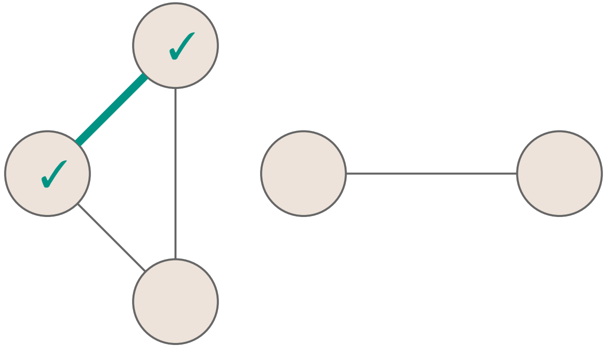

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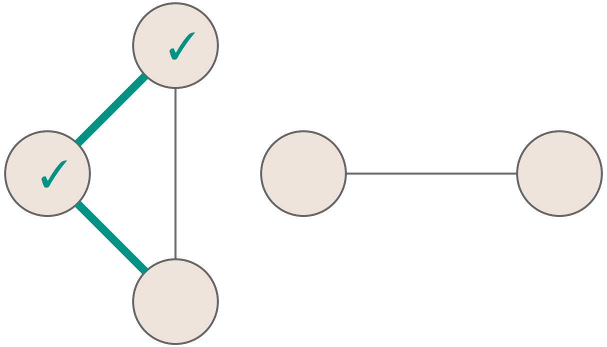

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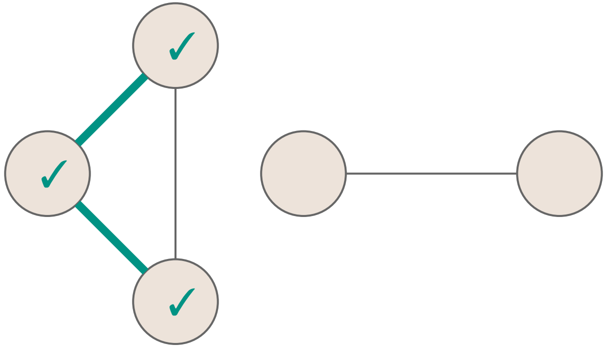

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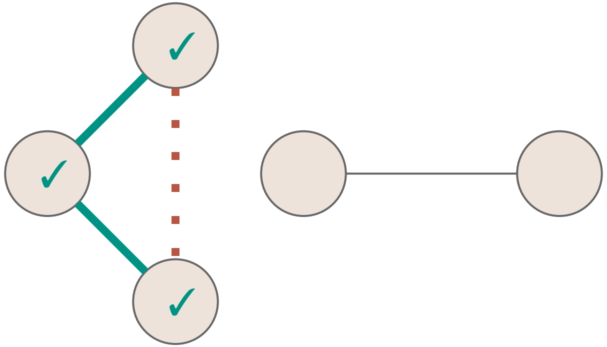

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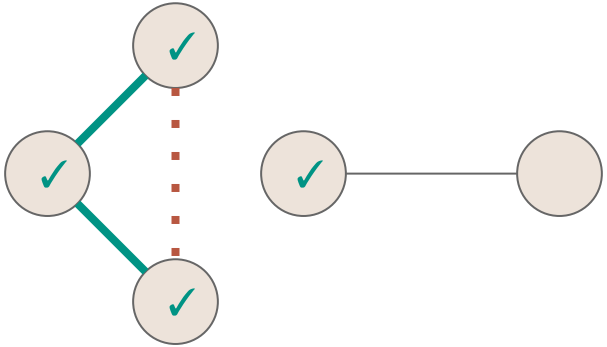

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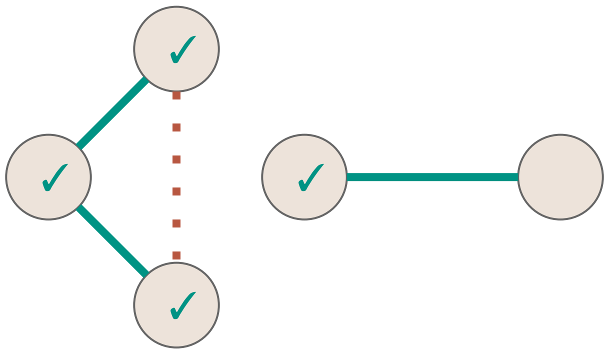

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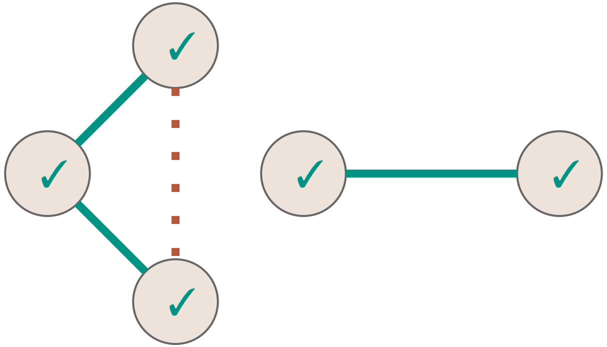

DFS

```
object VertexLabel extends Enumeration
  { val UNEXPLORED, VISITED = Value }

object EdgeLabel extends Enumeration
  { val UNEXPLORED, SPANNING, BACK = Value }

def DFS(graph: Graph[VertexLabel.Value, EdgeLabel.Value]) {
  for(v <- graph.vertices) { v.setLabel(VertexLabel.UNEXPLORED) }
  for(e <- graph.edges)    { e.setLabel(EdgeLabel.UNEXPLORED) }
  for(v <- graph.vertices) {
    if(v.label == VertexLabel.UNEXPLORED){
      DFSOne(graph, v)
    }
  }
}
```

DFSOne

```
def DFSOne(graph: Graph[…], v: Graph[…]#Vertex) {
  v.setLabel(VertexLabel.VISITED)

  for(e <- v.incident) {
    if(e.label == EdgeLabel.UNEXPLORED){
      val w = e.getOpposite(v)
      if(w.label == VertexLabel.UNEXPLORED){
        e.setLabel(EdgeLabel.SPANNING)
        DFSOne(graph, w)
      } else {
        e.setLabel(EdgeLabel.BACK)
      }
    }
  }
}
```

DFSOne

```
def DFSOne(graph: Graph[…], v: Graph[…]#Vertex) {
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        e.setLabel(EdgeLabel.BACK)
      }
    }
  }
}
```

If the edge is unexplored, explore it

DFSOne

```
def DFSOne(graph: Graph[…], v: Graph[…]#Vertex) {
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      val w = e.getOpposite(v)
      if(w.label == VertexLabel.UNEXPLORED){
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        DFSOne(graph, w)
      } else {
        e.setLabel(EdgeLabel.BACK)
      }
    }
  }
}
```

If the edge is unexplored, explore it

If the other endpoint is unexplored, this is a spanning edge, explore that vertex

DFSOne

```
def DFSOne(graph: Graph[…], v: Graph[…]#Vertex) {
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        e.setLabel(EdgeLabel.BACK)
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    }
  }
}
```

If the edge is unexplored, explore it

If the other endpoint is unexplored, this is a spanning edge, explore that vertex

If the other endpoint is already explored, this is a back edge

Detailed Example

UNEXPLORED

VISITED

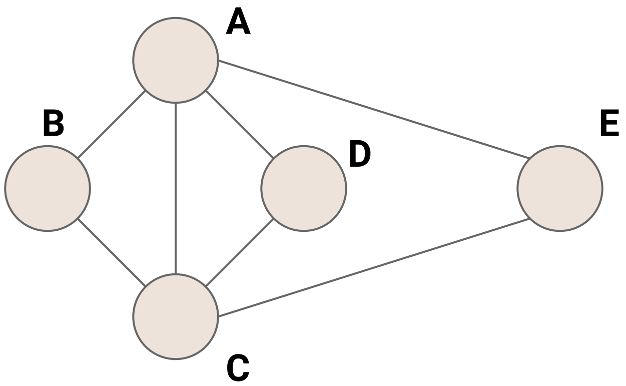
UNEXPLORED

SPANNING

BACK

Call Stack

(→ edges to list)



Detailed Example

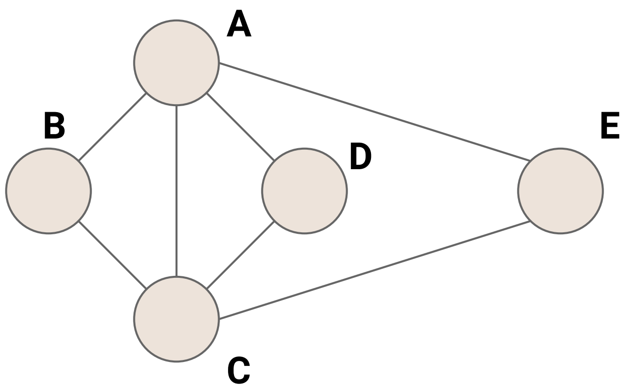
UNEXPLORED

✓ VISITED

UNEXPLORED

SPANNING

BACK



Call Stack

DFS(G)

(→ edges to list)

Detailed Example

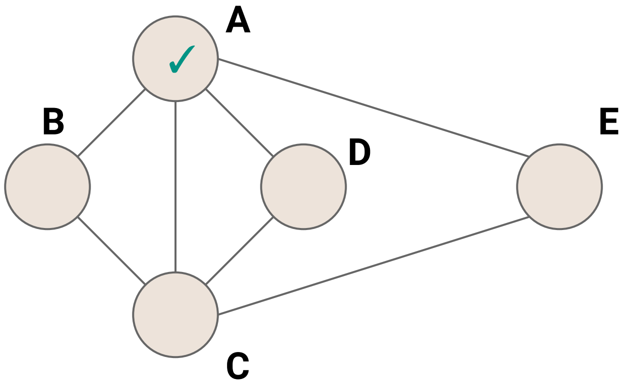
UNEXPLORED

✓ VISITED

UNEXPLORED

SPANNING

BACK



<u>Call Stack</u>

```
DFS(G)
DFSOne(G,A)
```

<u>(→ edges to list)</u>

Detailed Example

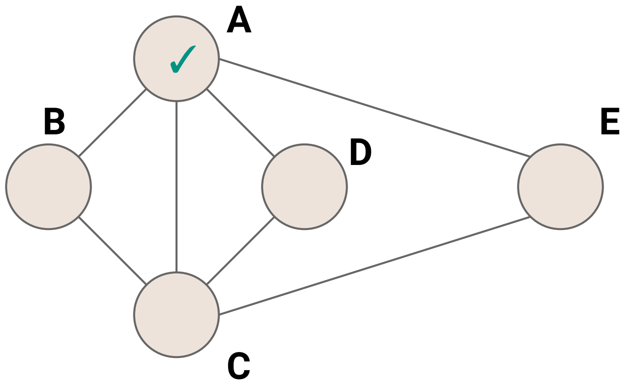
UNEXPLORED

VISITED

UNEXPLORED

SPANNING

BACK



Call Stack	(→ edges to list)
DFS(G)
DFSOne(G,A)	(→ B, C, D)

Detailed Example

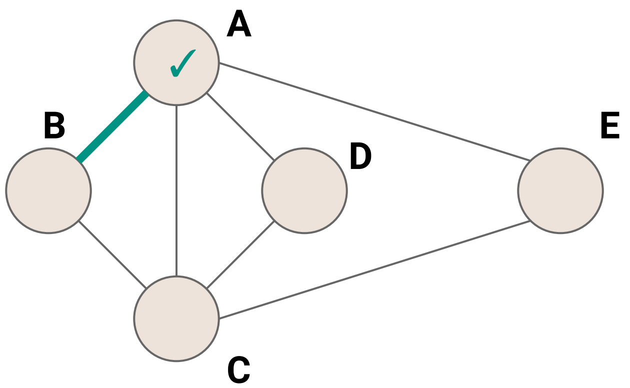
UNEXPLORED

✓ VISITED

UNEXPLORED

SPANNING

BACK



Call Stack	(→ edges to list)
DFS(G)
DFSOne(G,A)	(→ B, C, D)

Detailed Example

UNEXPLORED

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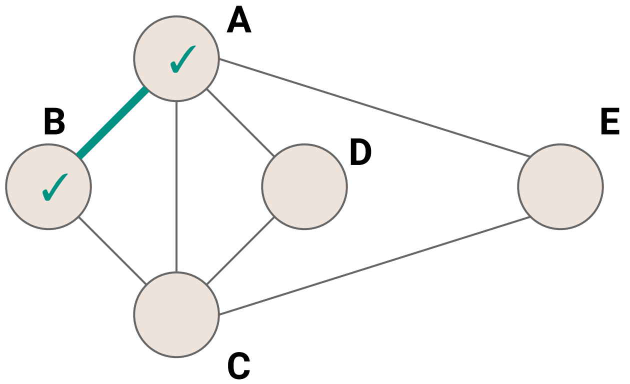
SPANNING

BACK



Call Stack	(→ edges to list)
`DFS(G)`	
`DFSOne(G,A)`	(→ B, C, D)
`DFSOne(G,B)`	(→ A, C)

Detailed Example

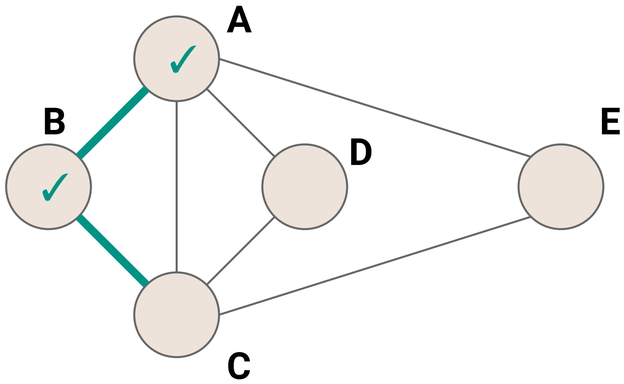
UNEXPLORED

✓ VISITED

UNEXPLORED

SPANNING

BACK



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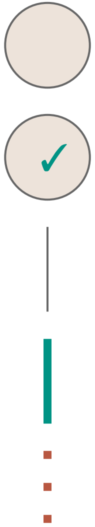
DFS(G)	
DFSOne(G,A)	(→ B, C, D)
DFSOne(G,B)	(→ A, C)

Detailed Example



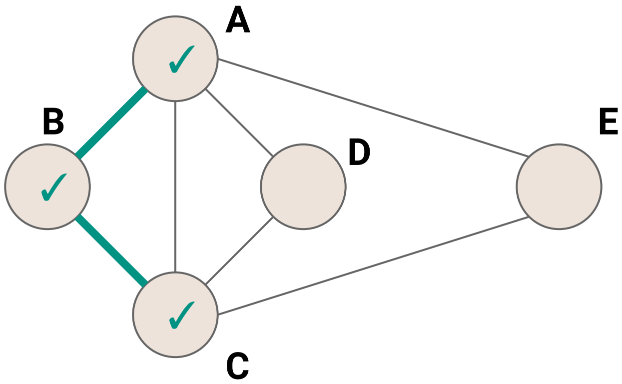
UNEXPLORED

VISITED

UNEXPLORED

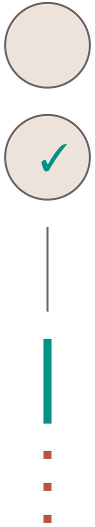
SPANNING

BACK



<u>Call Stack</u>	<u>(→ edges to list)</u>
DFS(G)	
DFSOne(G,A)	(→ B, C, D)
DFSOne(G,B)	(→ A, C)
DFSOne(G,C)	(→ B, A, D, E)

Detailed Example



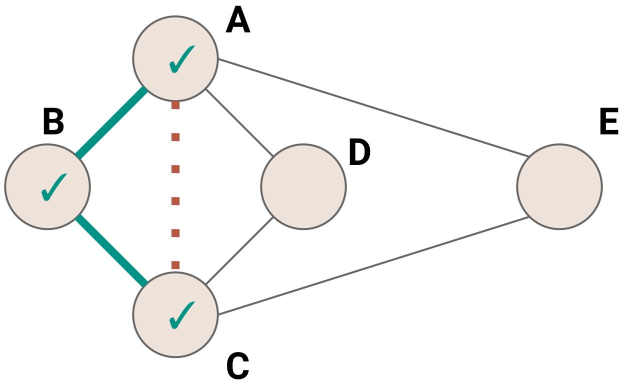
UNEXPLORED

VISITED

UNEXPLORED

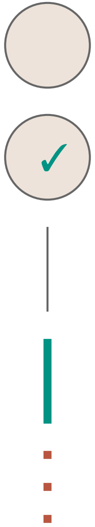
SPANNING

BACK



Call Stack	(→ edges to list)
`DFS(G)`	
`DFSOne(G,A)`	(→ B, C, D)
`DFSOne(G,B)`	(→ A, C)
`DFSOne(G,C)`	(→ B, A, D, E)

Detailed Example



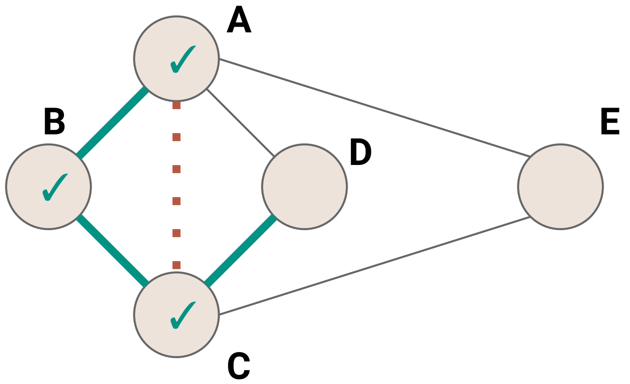
UNEXPLORED

VISITED

UNEXPLORED

SPANNING

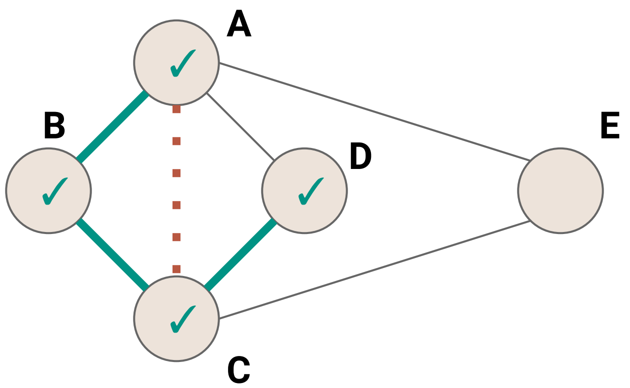
BACK



Call Stack	(→ edges to list)
DFS(G)	
DFSOne(G,A)	(→ B, C, D)
DFSOne(G,B)	(→ A, C)
DFSOne(G,C)	(→ B, A, D, E)

Detailed Example

- UNEXPLORED (node)
- VISITED (node)
- UNEXPLORED (edge)
- SPANNING
- BACK



<u>Call Stack</u>	<u>(→ edges to list)</u>
`DFS(G)`	
`DFSOne(G,A)`	(→ B, C, D)
`DFSOne(G,B)`	(→ A, C)
`DFSOne(G,C)`	(→ B, A, D, E)
`DFSOne(G,D)`	(→ A, C)

Detailed Example

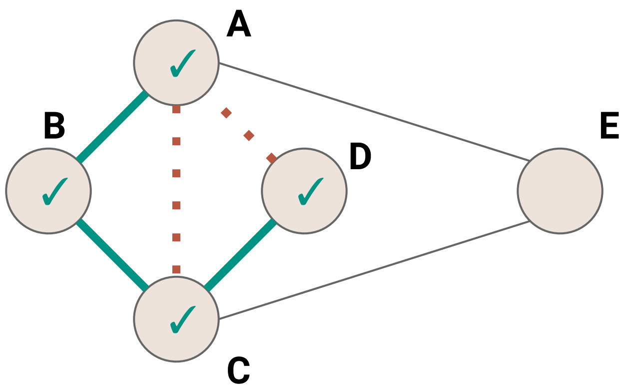
UNEXPLORED

VISITED

UNEXPLORED

SPANNING

BACK



Call Stack	(→ edges to list)
DFS(G)	
DFSOne(G,A)	(→ B, C, D)
DFSOne(G,B)	(→ A, C)
DFSOne(G,C)	(→ B, A, D, E)
DFSOne(G,D)	(→ A, C)

Detailed Example

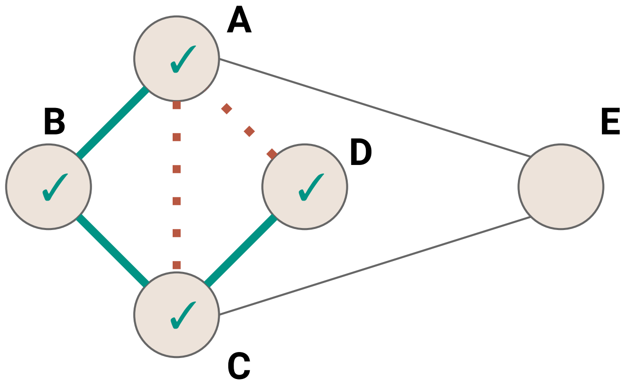
UNEXPLORED

VISITED

UNEXPLORED

SPANNING

BACK



Call Stack	(→ edges to list)
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DFSOne(G,A)	(→ B, C, D)
DFSOne(G,B)	(→ A, C)
DFSOne(G,C)	(→ B, A, D, E)

Detailed Example

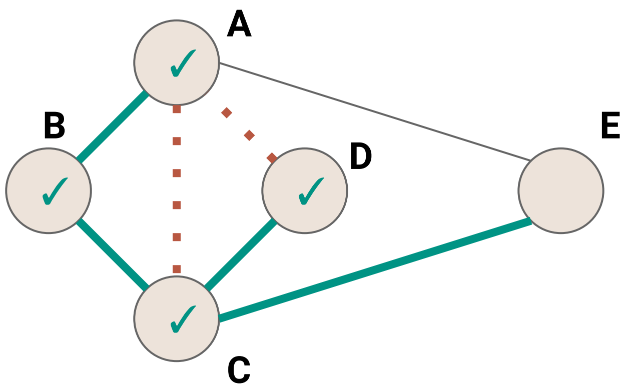
UNEXPLORED

VISITED

UNEXPLORED

SPANNING

BACK

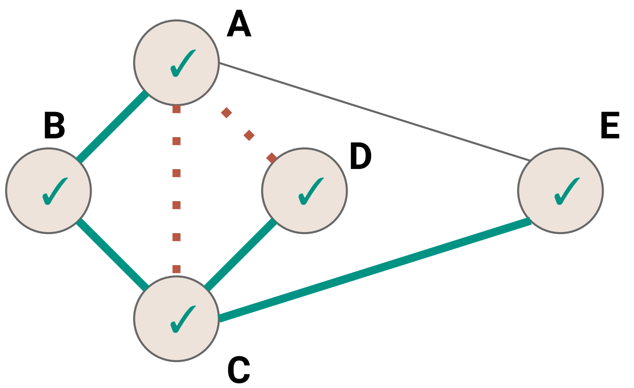


<u>Call Stack</u>	<u>(→ edges to list)</u>
DFS(G)	
DFSOne(G,A)	(→ B, C, D)
DFSOne(G,B)	(→ A, C)
DFSOne(G,C)	(→ B, A, D, E)

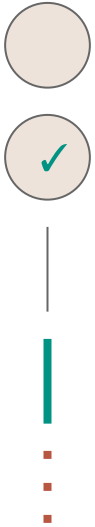
Detailed Example

- UNEXPLORED
- VISITED
- UNEXPLORED
- SPANNING
- BACK

<u>Call Stack</u>	<u>(→ edges to list)</u>
DFS(G)	
DFSOne(G,A)	(→ B, C, D)
DFSOne(G,B)	(→ A, C)
DFSOne(G,C)	(→ B, A, D, E)
DFSOne(G,E)	(→ A, C)



Detailed Example



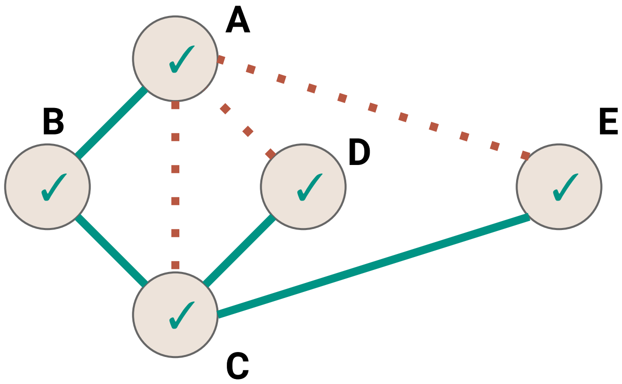
UNEXPLORED

VISITED

UNEXPLORED

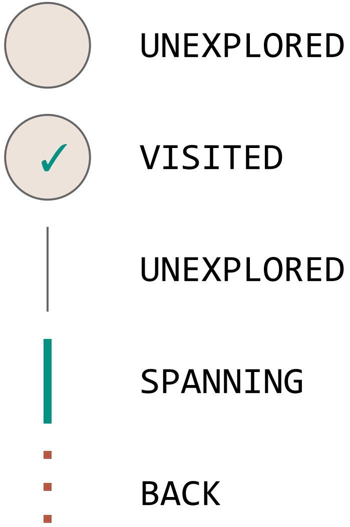
SPANNING

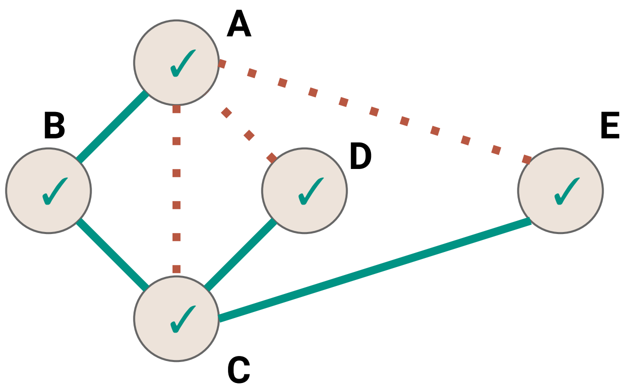
BACK



Call Stack	(→ edges to list)
DFS(G)	
DFSOne(G,A)	(→ B, C, D)
DFSOne(G,B)	(→ A, C)
DFSOne(G,C)	(→ B, A, D, E)
DFSOne(G,E)	(→ A, C)

Detailed Example





Call Stack	(→ edges to list)
DFS(G)	
DFSOne(G,A)	(→ B, C, D)
DFSOne(G,B)	(→ A, C)
DFSOne(G,C)	(→ B, A, D, E)

Detailed Example

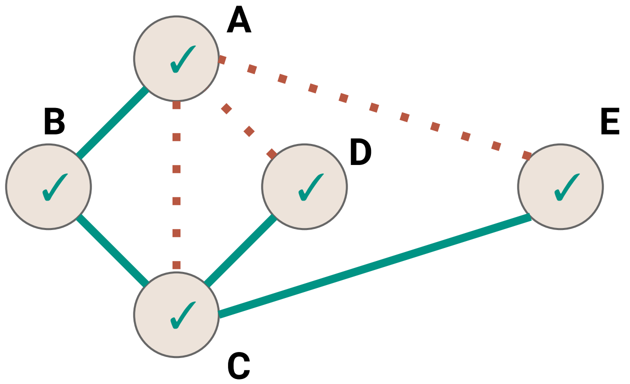
UNEXPLORED

VISITED

UNEXPLORED

SPANNING

BACK



Call Stack	(→ edges to list)
DFS(G)	
DFSOne(G,A)	(→ B, C, D)
DFSOne(G,B)	(→ A, C)

Detailed Example

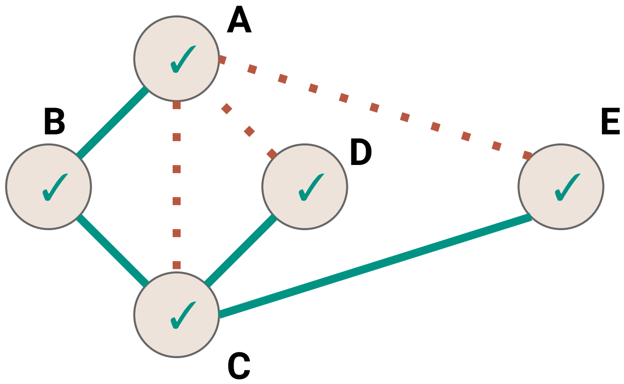
UNEXPLORED

VISITED

UNEXPLORED

SPANNING

BACK



<u>Call Stack</u>

DFS(G)

DFSOne(G,A)

<u>(→ edges to list)</u>

(→ B, C, D)

Detailed Example

UNEXPLORED

VISITED

UNEXPLORED

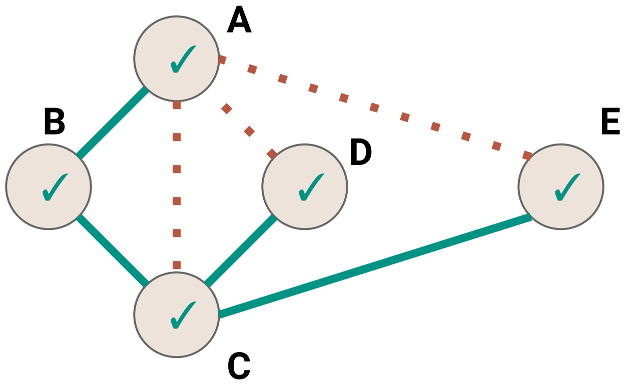
SPANNING

BACK

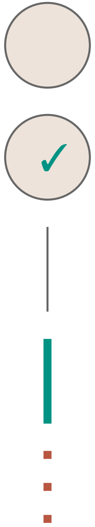
<u>Call Stack</u>

DFS(G)
DFSOne(G,B)

(→ edges to list)



Detailed Example



UNEXPLORED

VISITED

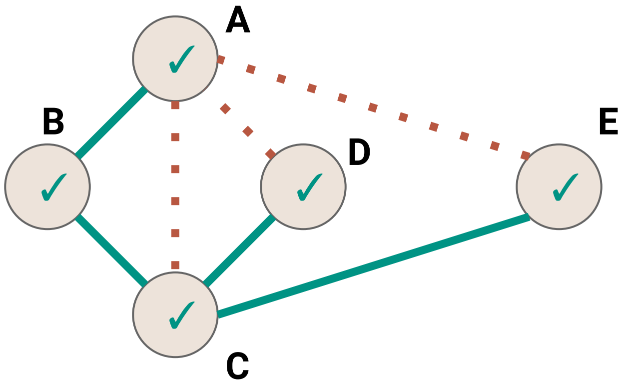
UNEXPLORED

SPANNING

BACK

<u>Call Stack</u>
DFS(G)
DFSOne(G,C)

(→ edges to list)



Detailed Example

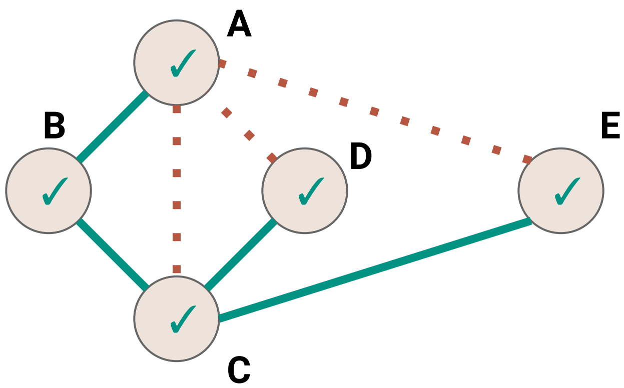
UNEXPLORED

VISITED

UNEXPLORED

SPANNING

BACK



Call Stack

DFS(G)

DFSOne(G,D)

(→ edges to list)

Detailed Example

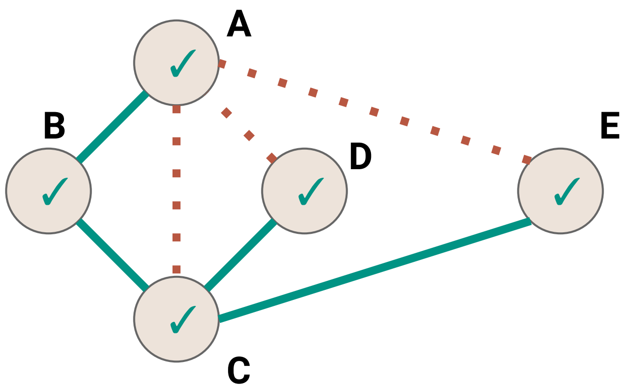
UNEXPLORED

VISITED

UNEXPLORED

SPANNING

BACK



<u>Call Stack</u>

```
DFS(G)
DFSOne(G,E)
```

(→ edges to list)

Detailed Example

UNEXPLORED

VISITED

UNEXPLORED

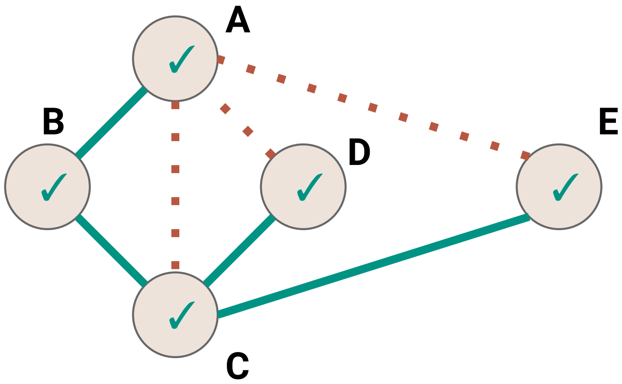
SPANNING

BACK

Call Stack

DFS(G)

(→ edges to list)



Detailed Example

UNEXPLORED

VISITED

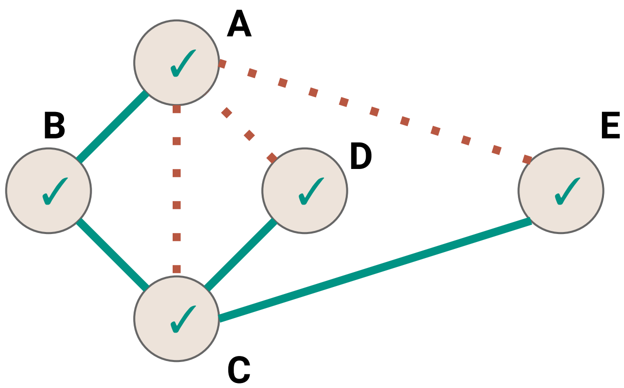
UNEXPLORED

SPANNING

BACK

Call Stack

(→ edges to list)



DFS vs Mazes

The DFS algorithm is like our stack-based maze solver

- Mark each grid square with **VISITED** as we explore it
- Mark each path with **SPANNING** or **BACK**
- Only visit each vertex once

DFS vs Mazes

The DFS algorithm is like our stack-based maze solver

- Mark each grid square with **VISITED** as we explore it
- Mark each path with **SPANNING** or **BACK**
- Only visit each vertex once
  - DFS will not necessarily find the shortest paths

Depth-First Search Complexity

What's the complexity?

Complexity

```
def DFS(graph: Graph[VertexLabel.Value, EdgeLabel.Value])
{
  for(v <- graph.vertices) { v.setLabel(VertexLabel.UNEXPLORED) }
  for(e <- graph.edges)    { e.setLabel(EdgeLabel.UNEXPLORED) }
  for(v <- graph.vertices) {
    if(v.label == VertexLabel.UNEXPLORED){
      DFSOne(graph, v)
    }
  }
}
```

Complexity

```
def DFS(graph: Graph[VertexLabel.Value, EdgeLabel.Value])
{
  /* O(|V|) */
  for(e <- graph.edges)     { e.setLabel(EdgeLabel.UNEXPLORED) }
  for(v <- graph.vertices) {
    if(v.label == VertexLabel.UNEXPLORED){
      DFSOne(graph, v)
    }
  }
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```

Complexity

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def DFS(graph: Graph[VertexLabel.Value, EdgeLabel.Value])
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  /* O(|V|) */
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      DFSOne(graph, v)
    }
  }
}
```

Complexity

```
def DFS(graph: Graph[VertexLabel.Value, EdgeLabel.Value])
{
  /* O(|V|) */
  /* O(|E|) */
  /* O(|V|) times */ {
    if(v.label == VertexLabel.UNEXPLORED){
      DFSOne(graph, v)
    }
  }
}
```

Complexity

```
def DFS(graph: Graph[VertexLabel.Value, EdgeLabel.Value])
{
  /* O(|V|) */
  /* O(|E|) */
  /* O(|V|) times */ {
    if(v.label == VertexLabel.UNEXPLORED){
      /* ??? */
    }
  }
}
```

Complexity

```
def DFSOne(graph: Graph[…], v: Graph[…]#Vertex) {
  v.setLabel(VertexLabel.VISITED)
  for(e <- v.incident) {
    if(e.label == EdgeLabel.UNEXPLORED){
      val w = e.getOpposite(v)
      if(w.label == VertexLabel.UNEXPLORED){
        e.setLabel(EdgeLabel.SPANNING)
        DFSOne(graph, w)
      } else {
        e.setLabel(EdgeLabel.BACK)
      }
    }
  }
}
```

Complexity

```
def DFSOne(graph: Graph[…], v: Graph[…]#Vertex) {
  /* O(1) */
  for(e <- v.incident) {
    if(e.label == EdgeLabel.UNEXPLORED){
      val w = e.getOpposite(v)
      if(w.label == VertexLabel.UNEXPLORED){
        e.setLabel(EdgeLabel.SPANNING)
        DFSOne(graph, w)
      } else {
        e.setLabel(EdgeLabel.BACK)
      }
    }
  }
}
```

Complexity

```
def DFSOne(graph: Graph[…], v: Graph[…]#Vertex) {
  /* O(1) */
  /* O(deg(v)) times */ {
    if(e.label == EdgeLabel.UNEXPLORED){
      val w = e.getOpposite(v)
      if(w.label == VertexLabel.UNEXPLORED){
        e.setLabel(EdgeLabel.SPANNING)
        DFSOne(graph, w)
      } else {
        e.setLabel(EdgeLabel.BACK)
      }
    }
  }
}
```

Complexity

```
def DFSOne(graph: Graph[…], v: Graph[…]#Vertex) {
  /* O(1) */
  /* O(deg(v)) times */ {
    /* O(1) */ {
      /* O(1) */
      /* O(1) */ {
        /* O(1) */
        DFSOne(graph, w)
      } else {
        /* O(1) */
      }
    }
  }
}
```

Complexity

```
def DFSOne(graph: Graph[…], v: Graph[…]#Vertex) {
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  /* O(deg(v)) times */ {
    /* O(1) */ {
      /* O(1) */
      /* O(1) */ {
        /* O(1) */
        /* ??? */
      } else {
        /* O(1) */
      }
    }
  }
}
```

Depth-First Search Complexity

How many times do we call **`DFSOne`** *on each vertex?*

Depth-First Search Complexity

How many times do we call **`DFSOne`** *on each vertex?*

Observation: **`DFSOne`** is called on each vertex *at most* once

If **`v.label == VISITED`**, both **`DFS`**, and **`DFSOne`** skip it

Depth-First Search Complexity

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$O(|V|)$ calls to **`DFSOne`**

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What's the runtime of **`DFSOne`** ***excluding the recursive calls?***

Complexity

```
def DFSOne(graph: Graph[…], v: Graph[…]#Vertex) {
  /* O(1) */
  /* O(deg(v)) times */ {
    /* O(1) */ {
      /* O(1) */
      /* O(1) */ {
        /* O(1) */
        /* ??? */
      } else {
        /* O(1) */
      }
    }
  }
}
```

Depth-First Search Complexity

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$O(|V|)$ calls to **`DFSOne`**

What's the runtime of **`DFSOne`** ***excluding the recursive calls?*** $O(\deg(v))$

Depth-First Search Complexity

What is the sum over all calls to `DFSOne`?

Depth-First Search Complexity

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$$\sum_{v \in V} O(deg(v))$$

Depth-First Search Complexity

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$$\sum_{v \in V} O(deg(v))$$

$$= O(\sum_{v \in V} deg(v))$$

Depth-First Search Complexity

What is the sum over all calls to **`DFSOne`**?

$$\sum_{v \in V} O(deg(v))$$

$$= O(\sum_{v \in V} deg(v))$$

$$= O(2|E|)$$

Depth-First Search Complexity

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$$\sum_{v \in V} O(deg(v))$$

$$= O(\sum_{v \in V} deg(v))$$

$$= O(2|E|)$$

$$= O(|E|)$$

Depth-First Search Complexity

In summary...

Depth-First Search Complexity

In summary...

1. Mark the vertices `UNVISITED`

Depth-First Search Complexity

In summary...

1. Mark the vertices **`UNVISITED`** $O(|V|)$

Depth-First Search Complexity

In summary…

1. Mark the vertices **`UNVISITED`** $O(|V|)$
2. Mark the edges **`UNVISITED`**

Depth-First Search Complexity

In summary...

1. Mark the vertices **`UNVISITED`** $O(|V|)$
2. Mark the edges **`UNVISITED`** $O(|E|)$

Depth-First Search Complexity

In summary…

1. Mark the vertices **`UNVISITED`** $O(|V|)$
2. Mark the edges **`UNVISITED`** $O(|E|)$
3. **`DFS`** vertex loop

Depth-First Search Complexity

In summary...

1. Mark the vertices **`UNVISITED`**	$O(\|V\|)$
2. Mark the edges **`UNVISITED`**	$O(\|E\|)$
3. **`DFS`** vertex loop	$O(\|V\|)$

Depth-First Search Complexity

In summary...

1.	Mark the vertices **`UNVISITED`**	$O(\|V\|)$
2.	Mark the edges **`UNVISITED`**	$O(\|E\|)$
3.	**`DFS`** vertex loop	$O(\|V\|)$
4.	All calls to **`DFSOne`**	

Depth-First Search Complexity

In summary...

1. Mark the vertices **`UNVISITED`** — $O(|V|)$
2. Mark the edges **`UNVISITED`** — $O(|E|)$
3. **`DFS`** vertex loop — $O(|V|)$
4. All calls to **`DFSOne`** — $O(|E|)$

Depth-First Search Complexity

In summary...

1. Mark the vertices **`UNVISITED`** — $O(|V|)$
2. Mark the edges **`UNVISITED`** — $O(|E|)$
3. **`DFS`** vertex loop — $O(|V|)$
4. All calls to **`DFSOne`** — $O(|E|)$

$$O(|V| + |E|)$$