Indexing

- Workflow Recap:
 - Load/Parse data records
 - Filter records (rank ≠ Ensign, age > 25)
 - Print names
- Today: Let's combine steps 1 & 2

- Basics: Sorting

- Root Idea: Sort records by age
 - Algorithm:
 - Can use Binary Search over data to find first record > 25
 - Return that record and everything following it
 - Challenges
 - Need a record format with predictable record locations
 - Fixed-Size Records
 - Put a fixed number of records on each "page"
 - Paging makes binary search pricy
 - What happens if the data changes?
 - Does this generalize?
 - age > X; Same as above
 - age < Y; Yes, Start from first record, return everything until first record >= X
 - age = X: Yes, Binary Search Still Works (may still need to return multiple records)
 - X < age < Y; Yes, Binary Search, then return everything until first record >= Y

Indexes

- Challenges
 - Paging (respectively cache lines) makes binary search expensive
 - Scan is still comparatively cheap
 - What if we need to access 2 (or more) attributes?
 - · Modulo a few corner cases, we can't sort more than once
 - No real answer for this point today... we'll get back to it
- Idea 1: Page-aware 'Key' Summaries
 - Implementation 1: One page of summaries
 - Fit as many [key+pointer] pairs as you can in one page
 - Each pointer points to the first record equal to or greater than the listed key
 - Binary search on keys to find the pointer to follow

- Limitation: Doesn't scale to larger data sizes; Still may need to binary search across data on multiple pages)
- Implementation 2: Add indirection (Tree Indexes)
 - Binary search within a page is cheap, so keep one [key+pointer] per page
 - Pack as many [key+pointer]s into a summary page as you can.
 - If you overflow the summary page, start building a summary of summaries
 - Tier 1: Data Pages
 - Tier 2: Pages of [Key+Pointer]s to the first key on each data page
 - Tier 3: Pages of [Key+Pointer]s to the first key on each tier 2 page
 - Tier 4: etc...
- Challenge: Handling Changing Data
 - Can't insert into the middle of a sorted file
 - · Can't insert into a packed (sorted) summary page
- Implementation 3: Out-of-order pages (B+Tree-Ish Indexes)
 - Treat pages as atomic blobs of storage (rather than a single contiguous region)
 - Bonus: Don't need fixed-size records
 - Leave empty space on each data page and each summary (tree) page
 - ▼ What to do when a page "fills up" or "empties out"?
 - · Shift records to/from other pages at the same level (pivot)
 - Merge two pages together
 - Create a new level / flatten a level
 - Degenerate case:
 - Super-tall structure
- ▼ Implementation 4: As above, but maintain size invariant (B+Tree)
 - Invariant 1: Uniform Tree Depth
 - **Invariant 2**: $50\% \le \text{fill} \le 100\%$ (for all except root page)
 - ▼ When page drops below 50% fill, merge with adjacent page
 - Recur higher if necessary
 - When page exceeds 100% fill, split into 2 pages
 - Recur higher if necessary
 - When root drops to 1 pointer, reduce depth by 1
 - When root exceeds capacity, increase depth by 1
 - Optimization: Borrow/Loan records/[key+pointer]s from/to adjacent pages